# United States Patent [19]

**Bartlett et al.**

[11] **3,737,909**

[45] **June 5, 1973**


[54] **PARABOLIC ANTENNA SYSTEM HAVING HIGH-ILLUMINATION AND SPILLOVER EFFICIENCIES**


[75] Inventors: **Homer E. Bartlett**, Melbourne, Fla.; **Emory L. Sheppard**, Morrisville, N.C.

[73] Assignee: **Radiation Incorporated**, Melbourne, Fla.



[22] Filed: **June 18, 1970**

[21] Appl. No.: **47,378**

[52] **U.S. Cl.** ....................**343/755**, 343/781, 343/840

[51] **Int. Cl.** ....................**H01q 19/10**

[58] **Field of Search** ....................343/753, 754, 755, 343/781, 840



[56] **References Cited**

UNITED STATES PATENTS

| | | | |
|---|---|---|---|
| 2,547,416 | 6/1951 | Skellett | 343/910 |
| 2,609,505 | 9/1952 | Pippard | 343/754 |
| 3,146,451 | 8/1964 | Sternberg | 343/910 |
| 3,456,260 | 7/1969 | Hannan | 343/755 |
| 3,550,139 | 12/1970 | McFarland | 343/754 |
| 2,705,753 | 4/1955 | Jaffe | 343/755 |
| 3,162,858 | 12/1964 | Cutler | 343/840 |
| 3,264,648 | 8/1966 | Sundberg et al. | 334/754 |
| 3,392,395 | 7/1968 | Hannan | 343/755 |
| 3,430,244 | 2/1969 | Bartlett et al. | 343/755 |

FOREIGN PATENTS OR APPLICATIONS

| | | | |
|---|---|---|---|
| 973,583 | 10/1964 | Great Britain | 343/840 |
| 1,163,156 | 9/1969 | Great Britain | 343/755 |



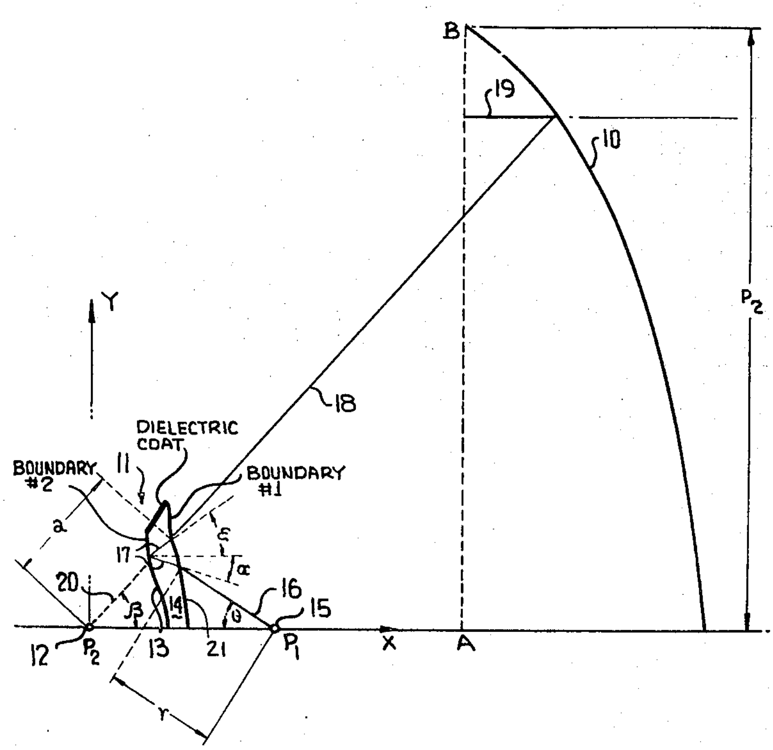
*Primary Examiner*—Eli Lieberman
*Attorney*—Yount and Tarolli



[57] **ABSTRACT**

An antenna feed system which simultaneously produces nearly uniform amplitude and phase illumination as well as high spillover efficiency, in a parabolic antenna, is composed of a feed or horn source and an interposed dielectric element. The dielectric element diffracts the emitted energy to maximize the spillover and illumination efficiencies. These efficiencies are increased by configuring the surface of the interposed dielectric element. In one species the interposed element is a lens and in another is a reflector coated with dielectric material.


**9 Claims, 5 Drawing Figures**



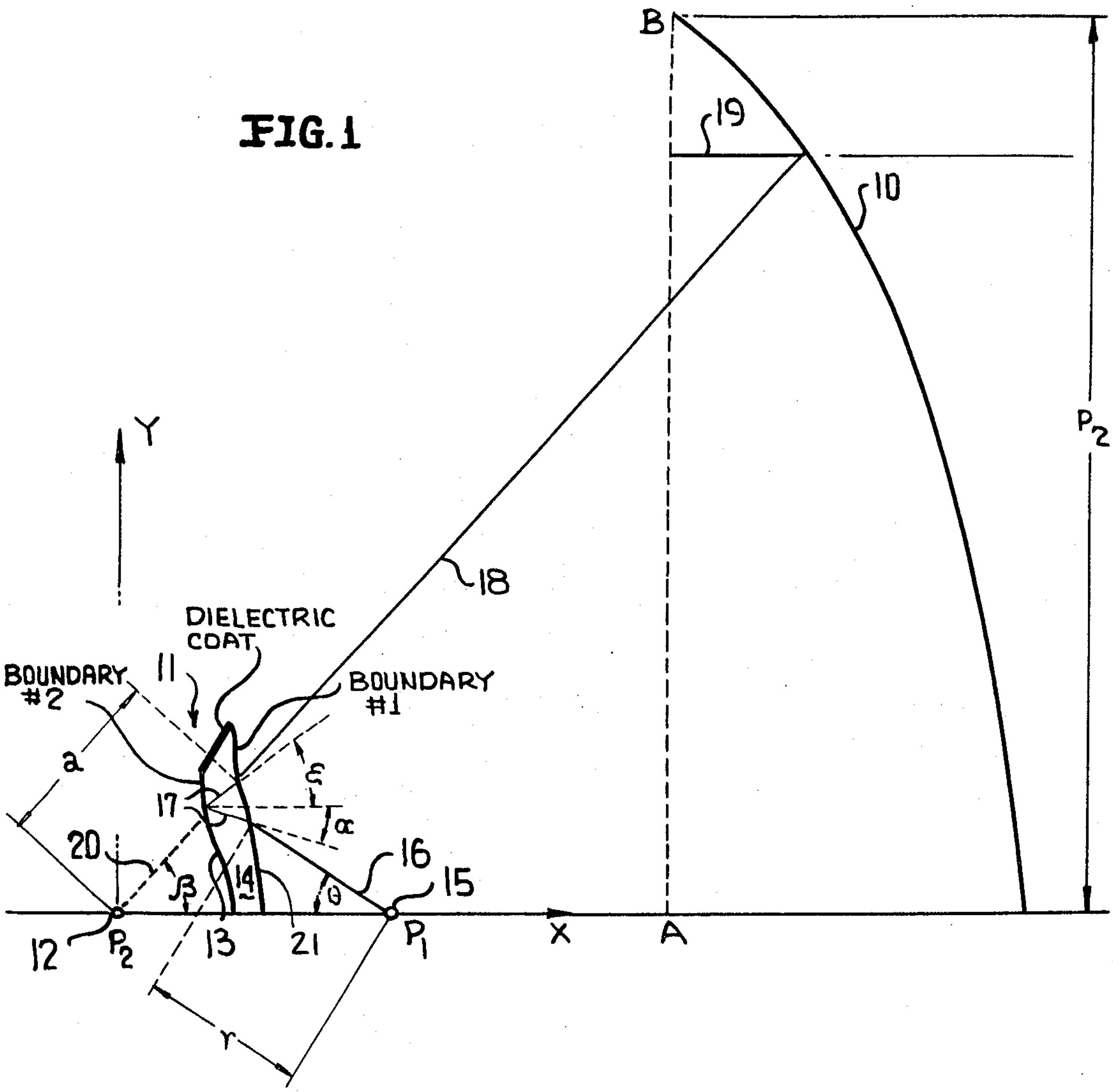
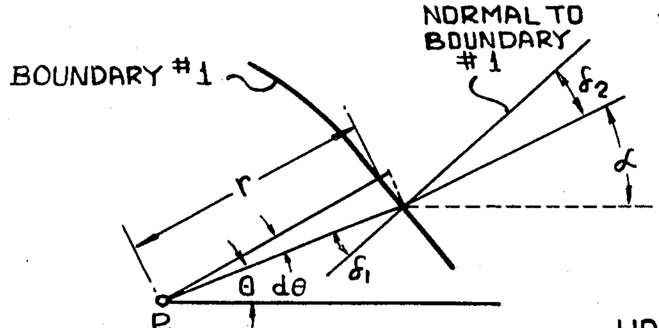



FIG. 1

FIG. 3




*INVENTORS*
HOMER E. BARTLETT
& EMORY L. SHEPPARD

By Hurvitz & Rose

ATTORNEYS

FIG.2

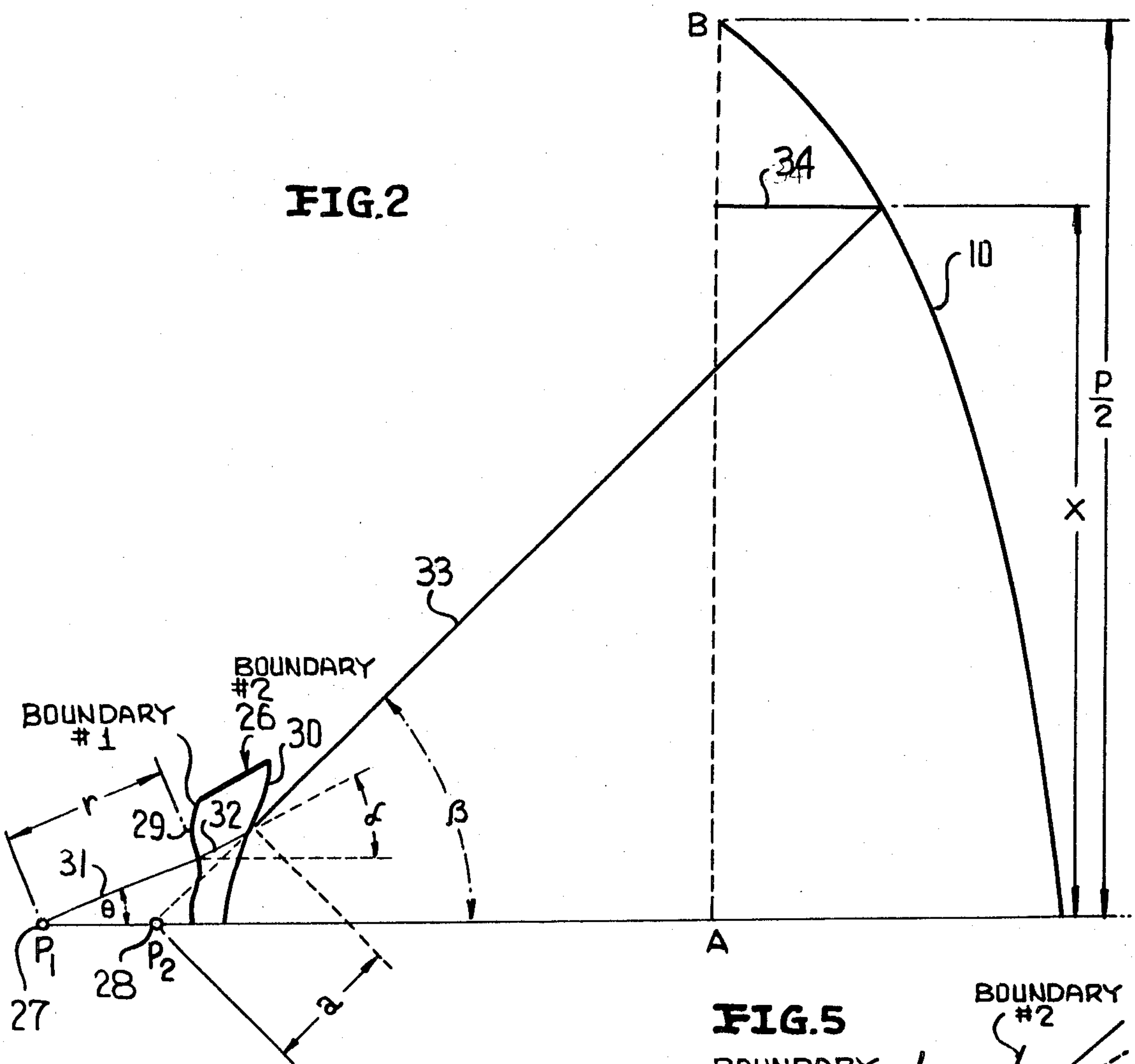



FIG.4

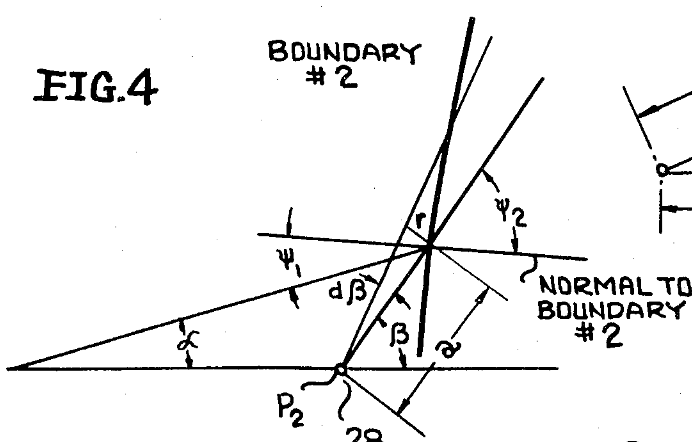
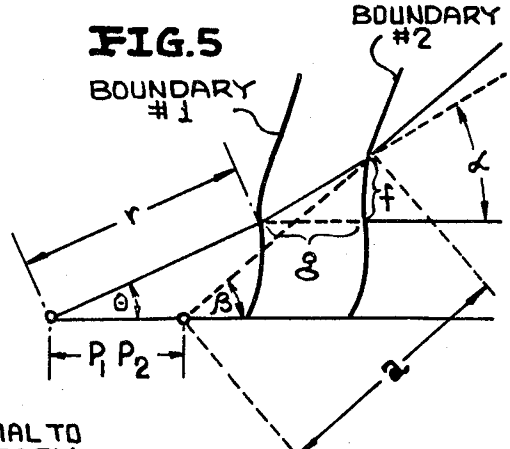



FIG.5




INVENTORS
HOMER E. BARTLETT
& EMDRY L. SHEPPARD

By Kurtz & Rose
ATTORNEYS

# PARABOLIC ANTENNA SYSTEM HAVING HIGH-ILLUMINATION AND SPILLOVER EFFICIENCIES

## BACKGROUND OF THE INVENTION

Various types of antenna systems are known in the prior art. One of these systems utilizes a reflecting element which is parabolically configured and which has a source of energy at its focal point. The energy emitted by the energy source is theoretically reflected from the parabolic reflector in substantially parallel rays. Although the following description is directed to a transmitting antenna system, the description is equally applicable to a receiving antenna.

Various attempts have been made to maximize the efficiency of such antenna systems. One such system is the Cassegrain. In this type of antenna system, a parabolic main reflector is used and a hyperbolic subreflector is located near the focal point of the parabolic main reflector. An energy source is then located near the hyperbolic subreflector. The energy emitted by the energy source is reflected by the subreflector onto the parabolic main reflector and out into the atmosphere. The over-all efficiency of such an antenna system is determined primarily by the ability of the energy source to illuminate the reflector uniformly across its surface. This is commonly referred to as the illumination efficiency. Another determining factor of the over-all efficiency of such an antenna system is the spillover efficiency. This is the ability of the subreflector to uniformly illuminate the main reflector while minimizing the energy which passes the edges of the main reflector.

To some extent these two efficiencies are inconsistent. This is so because the maximization of the illumination efficiency requires the edges of the main reflector to be illuminated to the same extent that its interior portions are illuminated. This inherently results in an increase of the radiation spillover and thereby decreases the spillover efficiency. Likewise, an attempt to decrease the energy spillover and thereby increase the spillover efficiency results in a decrease of the illumination along the edges of the reflector and a decrease in the illumination efficiency results. This difficulty has been recognized in the past and has been partially solved by compromising between the two efficiencies. Accordingly, most prior art antenna systems compromise between the illumination and the spillover efficiencies so that each are in the order of **75** to **80** per cent. These efficiencies are about the highest obtainable by conventional technology.

One known system has greatly improved these theoretical and practical efficiencies in a Cassegrain system such that a theoretical 100 percent illumination efficiency is obtainable while increasing the spillover efficiency to above 90 per cent. This system is based on the principle that planned deviations of the hyperbolic subreflector and parabolic main reflector from the hyperbolic and parabolic configurations can result in the above noted increases in efficiencies.

An article entitle "High Efficiency Antenna Reflector" by William F. Willaims, published in the July 1965 issue of The Microwave Journal on pages 79 to 82 describes an antenna system which improved the illumination and spillover efficiencies of a Cassegrain antenna system. The article presents a mathematical analysis showing that deviations of the sort mentioned above results in increases of the illumination and spillover efficiencies. Although the antenna system described in the article is a theoretical improvement of the other prior art antenna systems it has several practical disadvantages. As a practical matter, it is more expensive to manufacture a reflector which conforms to the required configuration. Also, the technique described does not allow for the optimization of antenna systems which are already in use.



## SUMMARY OF THE INVENTION

The inventive antenna system contains a dielectric refractive element to refract the energy emitted from the energy source so that the main reflector is uniformly illuminated over its entire surface while the energy spillover around its edge is minimized. These two results are achieved by forming the surface of the refracting element according to equations derived for an antenna system having a parabolic main reflector. The inventive system therefore is more practical than the system describe in the Microwave Journal article fully referenced hereinabove. Because a parabolic main reflector is used, existing manufacturing procedures can be used. For this reason the inventive antenna system is more economical and mechanically feasible than the prior art systems.

## BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a first preferred embodiment of the inventive system utilizing a reflective subreflector.

FIG. **2** is a second preferred embodiment of the inventive system utilizing a shaped lens which is transparent to the energy emitted by the energy source.

FIGS. **3**, **4** and **5** are diagrams useful in developing the equations which define the configurations of the refracting elements.

## DESCRIPTION OF THE PREFERRED EMBODIMENTS

The embodiment shown in FIG. **1** includes a reflector **10**, the cross-sectional configuration of which is parabolic. An energy refractor **11** is located close to the focal point **12** of the parabolic reflector **10.** Refractor **11** includes a reflective surface **13** and a dielectric coating **14.** The reflective surface **13** is made from a highly reflective metal. Dielectric coating **14** is made from a material which is transparent to the energy, polypropylene and quartz are examples of materials which can be used. An energy source **15** is located at a point $P_1$ positioned between refractor **11** and parabolic reflector **10.** Solid lines **16, 17, 18,** and **19** are used to illustrate the path of radiation from the energy source **15** through the dielectric coat **14** and to the reflector **10** out to the atmosphere from the reflector. Broken line **20** is used as an extension of line **18** to show that the radiation appears to emanate from the focus point **12** located at a point $P_2$. Only half of the parabolic reflector **10** is shown in the figure; actually, the reflector **10** is symmetrical about the X axis and also has radial symmetry about this axis. Reflector **11** also is radially symmetrical about the X axis. Reflector **11** has two irregular surfaces. Reflective element **13** is bonded, or otherwise fixed to one of these surfaces. The other surface is indicated by reference number **21.** Both of these surfaces are irregular, and they can have different configurations. The exact configurations are defined by a set of equations which are based on the parameters of the antenna system, and the dielectric constant of coating **14**. The illumination efficiency is optimized and the spillover efficiency is greatly increased by configuring the two surfaces of the refractor **11** in accordance with these equations and their development is presented hereinafter.

FIG. **2** is a second preferred embodiment of the invention. This embodiment also employs a parabolic reflector **10.** A shaped lens **26,** which is transparent to the radiation emitted by the energy source **27,** is located in the vicinity of the focal point **28** of the parabolic reflector **10.** Lens **26** is composed of a dielectric material such as quartz or polypropylene. The two irregular surfaces **29** and **30** of lens **26** are configured such that the energy passing through the lens optimizes the illumination efficiency of the reflector **10.** The configurations required for this optimization are also defined by a set of equations based on the same types of considerations as those of the FIG. **1** embodiment.

The FIG. **2** solid lines **31, 32, 33** and **34** are used to show the path of the radiation from the feed source **27** through the lens **26,** its path between refractor **26** and reflector **10,** and its reflection from the reflector **10.** In both FIGS. 1 and 2 the line A–B is used to represent a surface upon which constant amplitude and phase illumination is desired.

The primary difference between the FIG. 1 and FIG. 2 embodiments lies in its refractive elements **11** and **26.** In FIG. **1** a reflective surface **13** reflects the energy back through the dielectric **14.** Consequently energy source **15** is positioned between refractor **11** and reflector **10.** In FIG. **2** refractor **26** has no reflective surface. Energy therefore passes through the dielectric lens **26** only once. Lens **26** is therefore positioned between energy source **27** and reflector **10.**

The refractive elements **11** and **26** are very similar, in that they bend the energy after it emanates from the energy source but before it reaches parabolic reflector **10.** They are also similar because they both have two irregular surfaces defined by equations developed by using the same system criterion. Lens **26** and refractor **11** are symmetrical about the X axis and their axis of symmetry are coincident with the axis of symmetry or reflector **10.** The mean thickness of lens **26** and refractor **11** can be as thin as a fraction of an inch and can exceed 5 inches.

Both refractive elements **11** and **26** respectively shown in FIGS. **1** and **2** are designed such that the percentage of the energy from the energy source which is contained within the solid angle $\theta$ is equal to the percentage of the aperture area contained within a circle of radius X. The aperture illumination is therefore very nearly uniform. Also, the design configurations of the refractive elements **11** and **26** are such that the energy leaving the dielectric refractor (**11** or **26**) must have a spherical phase front about point $P_2$.

The result of shaping the surfaces of lens **26** and refractor **11** as dictated by the equations set forth hereinafter invariably will result in the surfaces being irregular in configuration. However, the configurations shown in the figures are not necessarily those which will be derived for every instance. The shapes of the surfaces will be dependent upon the design parameters of the antenna system as well as the dielectric constant of the material used to construct the refractive element **11** or **26.**

The following four equations described the configurations of surfaces **29** and **30** of lens **26** shown in the FIG. **2** embodiment

$$-(dr/rd\theta) = \sin(\theta-\alpha)/\left(\frac{1}{\sqrt{\epsilon}}\right) - \cos(\theta-\alpha) \tag{1}$$

$$da/ad\beta = \sqrt{\epsilon}\ \sin(\beta-\alpha)\ \sqrt{\epsilon}\ \cos(\beta-\alpha) - 1 \tag{2}$$

$$\frac{d\beta}{d\theta} = \frac{(1-\cos\beta_m)F(\theta)\sin\theta}{\sin\beta\int_0^\theta F(\theta)\sin\theta d\theta} \tag{3}$$

$$\tan\alpha = (a\sin\beta - r\sin\theta)/(P_1P_2 + a\cos\beta - r\cos\theta) \tag{4}$$

where: $\epsilon$ is the dielectric constant of the dielectric lens **26,** all other variables are defined by FIG. **2.**

In equation (3) the maximum value of $\beta$ is $\theta$by $\beta_m$ and represents the angle from the edge of the parabolic reflector **10.** Likewise, $\theta_m$ is the maximum value of **18** at which an electromagnetic ray from $P_1$ will be refracted by the lens **26** at an angle $\beta_m$ to the edge of the parabolic reflector **10.**

Equation (1) is written with Snell's Law of Refraction at surface **29.** It is used to relate the slope of surface **29** to the angles $\theta$ and $\alpha$.

Equation (2) is written with Snell's Law of Refraction at surface **30.** It is used to relate the angles $\beta$ and $\alpha$ to the slope of surface **30.** It also requires the energy leaving the lens **26** to have a constant phase about point $P_2$. $P_2$ is the focal point of parabolic reflector **10.**

$P_1$ is the origin of the polar coordinate system r,$\theta$ and also the phase center of the feed source **27.**

In equation (4) $P_1 P_2$ is the distance between points $P_1$ and $P_2$.

A better understanding of equations (1) – (4) can be obtained by viewing their development.

Equation (1) is developed by use of Snell's Law of Refraction at surface **29.** For convenience and clarity those portions of FIG. **2** required for the development of equation (1) are shown in FIG. **3.**

From FIG. **3**:

$$\alpha = \theta + \delta_1 - \delta_2 \tag{1a}$$

$\delta_1 \neq \delta_2$ because of the dielectric constant $\epsilon$ of the lens material

$$\sqrt{\epsilon}\sin\delta_2 = \sin\delta_1 \tag{1b}$$

$$-(dr/rd\theta) = \tan\gamma = \tan\delta_1 \tag{1c}$$

$$\sqrt{\epsilon}\sin(\theta + \delta_1 - \alpha) = \sin\delta_1 \tag{1d}$$

$$\sin(\theta-\alpha)\cos\delta_1 + \cos(\theta-\alpha)\sin\delta_1 = \sin\delta_1/\sqrt{\epsilon} \tag{1e}$$

$$\cot\delta_1 = [(1\sqrt{\ \ \epsilon}) - \cos(\theta-\alpha)]/\sin(\theta-\alpha) \tag{1f}$$

and

$$-(dr/rd\theta) = \sin(\theta-\alpha)/(1\sqrt{\epsilon}) - \cos(\theta-\alpha) \qquad (1)$$

Equation (2) is developed with Snell's Law of Refraction at surface **30.** Here, for purposes of convenience and clarity those portions of FIG. **2** required for the development of equations (2) are repeated in FIG. **4.**

From FIG. **4**:

$$da/ad\beta = \tan\gamma = \tan(\Psi_1 + \beta - \alpha) \qquad (2a)$$

$$\sqrt{\epsilon}\sin\Psi_1 = \sin\Psi_2 \qquad (2b)$$

$$\sqrt{\epsilon}\sin\Psi_1 = \sin(\beta-\alpha+\Psi_1) = \sin(\beta-\alpha)\cos\Psi_1 + \cos(\beta-\alpha)\sin\Psi_1 \qquad (2c)$$

$$\sqrt{\epsilon} = \cot\Psi_1 \sin(\beta-\alpha) + \cos(\beta-\alpha) \qquad (2d)$$

$$\cot\Psi_1 = [\sqrt{\epsilon} - \cos(\beta-\alpha)]/\sin(\beta-\alpha) \qquad 2e)$$

$$\tan\Psi_1 = \sin(\beta-\alpha)/[\sqrt{\epsilon} - \cos(\beta-\alpha)] \qquad (2f)$$

$$da/ad\beta = [\tan\Psi_1 + \tan(\beta-\alpha)]/[1 - \tan\Psi_1 \tan(\beta-\alpha)] \qquad (2g)$$

and

$$da/ad\beta = \sqrt{\epsilon}\sin(\beta-\alpha)/\ \sqrt{\epsilon}\cos(\beta-\alpha) - 1 \qquad (2)$$

Equation (3) is derived from the conservation of energy principle as follows:

$$\int_0^{\theta_m} F(\theta)\sin\theta d\theta = \int_0^{\beta_m} I(\beta)\sin\beta d\beta \qquad (3a)$$

$$\int_0^{\theta} F(\theta)\sin\theta d\theta = \int_0^{\beta} I(\beta)\sin\beta d\beta \qquad (3b)$$

Integration of the right side of equations (3*a*) and (3*b*) while holding $I(\beta)$ = constant, subsequent division of (3*a*) by (3*b*) and their differentiation of the result with respect to $\theta$ yields:

$$\frac{d\beta}{d\theta} = \frac{(1-\cos\beta_m)F(\theta)\sin\theta}{\sin\beta\int_0^{\theta_m} F(\theta)\sin\theta d\theta} \qquad (3)$$

FIG. **5** combines the portions of FIGS. **3** and **4** necessary to an understanding of equation (4).

From FIG. **5**:

$$\tan\alpha = f/g \qquad (4a)$$

$$f = a\sin\beta - r\sin\theta \qquad (4b)$$

$$g = P_1P_2 + a\cos\beta - r\cos\theta \qquad (4c)$$

$$\tan\alpha = (a\sin\beta - r\sin\theta)/(P_1P_2 + a\cos\beta - r\cos\theta) \qquad (4)$$

FIG. **2** and equations (1) – (4) related thereto are directed to an antenna utilizing a transparent lens **26** to optimize the illumination efficiency. FIG. **1** shows an antenna system which utilizes a dielectric coated subreflector as the energy refracting device. The geometry of the two antenna systems therefore varies slightly. However, the same principles apply.

Equations (5) through (10) are presented below as those defining the boundary surfaces **13** and **21** of the FIG. **1** embodiment necessary to optimize the illumination efficiency of an antenna utilizing a dielectric coated subreflector.

$$dr/rd\theta = \sqrt{\epsilon}\sin(\theta-\alpha)/\ \sqrt{\epsilon}\cos(\theta-\alpha) - 1 \qquad (5)$$

$$da/ad\beta = \sqrt{\epsilon}\sin(\beta-\xi)/\ \sqrt{\epsilon}\cos(\beta-\xi) - 1 \qquad (6)$$

$$\frac{d\beta}{d\theta} = \frac{(1-\cos\beta_m)F(\theta)\sin\theta}{\sin\beta\int_0^{\theta_m} F(\theta)\sin\theta d\theta} \qquad (7)$$

$$dx/dy = -\tan(\xi - \alpha)/2 \qquad (8)$$

$$\alpha = \tan^{-1}(y - r\sin\theta)/(P_1P_2 - x - r\cos\theta) \qquad (9)$$

$$\xi = \tan^{-1}(a\sin\beta - y)/(a\cos\beta - x) \qquad (10)$$

Equations (5) and (6) are developed by use of Snell's Law of Refraction at surface **21** in a manner very similar to Equations (1) and (2). The symbol $\epsilon$ is the dielectric constant of coating **14.**

Equation (5) relates the slope of surface **21** to the angles $\theta$ and $\alpha$. Equation (6) relates the slope of surface **13** to the angles $\beta$ and $\xi$.

Equation (7) is the same as Equation (3). Some manipulation of this equation shows that the energy density as a function of $\beta$ is $\sec^4 \beta/2$ for $0 < \beta < \beta_m$ and zero for $\beta > \beta_m$. Uniform amplitude distribution at Plane AB results from this relationship.

Equation (8) is developed by use of Snell's Law of Refraction at surface **13** and relates the slope of surface **13** to the angles $\alpha$ and $\xi$.

Equations (9) and (10) relate the points of surface **13** to those of surface **21.** In equations (9) and (10) $x$ and $y$ respectively refer to the horizontal and vertical coordinates of a coordinate system having its origin at $P_2$.

Because the same basic principles apply to equations (1) – (4) and equations (5) through (10) the full development of the last set of equations is not presented.

The problem of optimizing the illumination efficiency of a parabolic antenna system while simultaneously increasing the spillover efficiency has been solved by the invention. The solution lies in providing the antenna system with an energy refraction device interposed between the energy source and the main parabolic reflector. The two surfaces of the refracting device are irregularly formed according to a set of equations based upon the parameters of the antenna system. The equations are developed for an antenna system having a parabolic reflector and therefore the invention is very practical from both a theoretical and a manufacturing view point. The use of a parabolic main reflector also makes antenna systems in accordance with the invention and thereby greatly improve their illumination and spillover efficiencies.

While we have described and illustrated specific embodiments of our invention, it will be clear that variations of the details of construction which are specifically illustrated and described may be resorted to without departing from the true spirit and scope of the invention as defined in the appended claims.

We claim:

**1.** An antenna system having a main focusing means for redirecting energy having a focus proximate the axis of symmetry of said main focusing means and a feed device having a phase center proximate the axis of symmetry, said antenna system including a dielectric energy refracting means through which a beam transmitted between said main focusing means and said feed device passes, said refracting means having a first irregular surface means at which energy is redirected and substantially wholly comprising segments whose slopes are each a function of beam intensity at the segment to redirect the beam across an intervening space to a boundary at which the intensity of the beam has a relative distribution within the beam different from that of the beam at the first surface means, and a second irregular surface means at said boundary comprising a plurality of segments having slopes which are a function of the directions of the redirected beam at the segments for further redirecting the beam to proceed along a course comprising lines extending from one of said focus and center, said slopes of the segments of one of said irregular surface means of said refracting means being a function of a first angle between the axis of symmetry of said main focusing means and the apparent line of travel of energy through said focus and a second angle formed between the axis of symmetry and the line of travel of said energy through said refracting means, and said slopes of the segments of the other of said irregular surface means being a function of said second angle and of a third angle between the axis of symmetry and the apparent line of travel of energy through said phase center.

**2.** The antenna system of claim **1** wherein said energy refracting means is a transparent lens; and the two surface means of the lens are defined by the equations:

$$-(dr/rd\theta) = \sin(\theta-\alpha)/(\frac{1}{\sqrt{\epsilon}}) - \cos(\theta-\alpha) \quad (1)$$

$$da/ad\beta = \sqrt{\epsilon}\sin(\beta-\alpha)/\sqrt{\epsilon}\cos(\beta-\alpha) - 1 \quad (2)$$

$$\frac{d\beta}{d\theta} = \frac{(1-\cos\beta_m)F(\theta)\sin\theta}{\sin\beta\int_0^{\theta_m}F(\theta)\sin\theta\, d\theta} \quad (3)$$

$$\tan\alpha = (a\sin\beta - r\sin\theta)/(P_1 P_2 + a\cos\beta - r\cos\theta) \quad (4)$$

wherein:

- a = the distance from said focus to said one surface means
- $\epsilon$ = the dielectric constant of said refracting means
- $\beta$ = said first angle
- $\beta_m$ = maximum value of $\beta$
- $\alpha$ = said second angle
- $r$ = the distance from said phase center to said other of said surface means
- $\theta$ = said third angle
- $\theta_m$ = maximum value of $\theta$.

**3.** The antenna system of claim **1** wherein said refracting means is located between said feed device and said main focusing means; and wherein substantially all of the energy coupled to said feed device passes through said refracting means.

**4.** An antenna system as defined in claim **1** and wherein a relative distribution of intensity of the reflected beam resulting external to the antenna is substantially uniform, and wherein said beam has a first phase front about said focus and has a second phase front about said phase center and at least one of said first and second phase fronts is approximately spherical.

**5.** An antenna system having a main focusing means for redirecting energy having a focus proximate the axis of symmetry of said main focusing means and a feed device having a phase center proximate the axis of symmetry, said antenna system including a dielectric energy refracting means through which a beam transmitted between said main focusing means and said feed device passes, said refracting means having a first irregular surface means at which energy is redirected and substantially wholly comprising segments whose slopes are each a function of beam intensity at the segment to redirect the beam across an intervening space to a boundary at which the intensity of the beam has a relative distribution within the beam different from the beam redirected by the first surface means, and a second irregular surface means at said boundary comprising a plurality of segments having slopes which are a function of the directions of the redirected beam at the segments for further redirecting the beam to proceed along a source comprising lines extending from one of said focus and center, said refracting means including highly reflective means on one of said irregular surface means, said refracting means being arranged so that energy in said beam passes in a first direction through the other of said surface means and through said refracting means and impinges upon said reflective means and passes through aid refracting means in a second direction between said reflective means and through said other surface means, said slopes of said other of said surface means being a function of a first angle formed between said axis of symmetry of the main focusing means and the line of travel of said energy along lines extending from the focal point of said main focusing means and a second angle formed between said axis and the line of travel of said energy as it passes through said refraction means, said slopes of the one of said surface means having said reflective means being a function of said second angle and a third angle formed between said axis and the line of travel of said energy as it passes in said second direction through said refracting means; and said slopes of the other of said surface means of said refracting means being a function of said third angle and a fourth angle formed between the axis of symmetry of said main focusing means and the line of travel of said energy along lines extending from said phase center.

**6.** the antenna system of claim **5** wherein said reflective means on said one surface means and said other surface means are defined by the equations:

$$dr/rd\theta = \sqrt{\epsilon}\,\sin(\theta-\alpha)/\ \sqrt{\epsilon}\,\cos(\theta-\alpha) - 1 \qquad (5)$$

$$da/ad\beta = \sqrt{\epsilon}\,\sin(\beta-\xi)/\ \sqrt{\epsilon}\,\cos(\beta-\xi) - 1 \qquad (6)$$

$$\frac{a\beta}{a\theta} = \frac{(1-\cos\beta_m)\ F(\theta)\ \sin\theta}{\sin\beta \int_0^{\theta_m} F(\theta)\ \sin\theta d\theta} \qquad (7)$$

$$dx/dy = -\tan(\xi-\alpha)/2 \qquad (8)$$

$$\alpha = \tan^{-1}(y - r\sin\theta)/P_1P_2 - x - r\cos\theta) \qquad (9)$$

$$\xi = \tan^{-1}(a\sin\beta - y)/(a\cos\beta - x) \qquad (10)$$

wherein:

$\epsilon$ = the dielectric constant of said refracting means
$a$ = the distance from said focus to said other of said surfaces along said lines extending from said focal point
$\beta$ = said first angle
$\beta_m$ = maximum value of $\beta$
$\xi$ = said second angle
$\theta$ = said fourth angle
$\theta_m$ = maximum value of $\theta$
$\alpha$ = said third angle
$r$ = the distance from said phase center to said other surface along said lines extending from said phase center
$x$ = a coordinate of a rectangular coordinate system having its origin at said first focal point, extending along said axis
$y$ = a coordinate of said coordinate system extending transversely to said axis.

**7.** The antenna system of claim **1** wherein said highly reflective means is a metallic surface; and wherein said feed device is located between said refracting means and said main focusing means; said refracting means being situated between said metallic surface and said feed device.

**8.** An antenna system as defined in claim **5** and wherein a relative distribution of intensity of the reflected beam resulting external to the antenna is substantially uniform, and wherein said beam has a first phase front about said focus and has a second phase front about said phase center and at least one of said first and second phase fronts is approximately spherical.

**9.** An antenna apparatus for transmitting or receiving electromagnetic power comprising main focusing means for redirecting power with a principal axis and having a geometric focus, and a feed device having a phase center and a power density pattern $F(\theta)$, comprising lens means having first and second boundary portions for changing the direction of propagation of electromagnetic power received thereon, the power between said phase center and said main focusing means being a beam having a first segment at least a portion of which is between said phase center and lens means and a second segment at least a portion of which is between said lens means and said main focusing means, said first and second boundaries extending generally transversely of the axis and said second boundary being offset a distance along the beam route from the first means in a downstream direction for power flow, said first beam segment being directed along lines emanating from said phase center at various directional angles $\theta$ up to an angle $\theta_m$, said second beam segment being directed along lines emanating from said focus at various directional angles $\beta$ up to an angle $\beta_m$, both $\theta$ and $\beta$ being measured from the principal axis, said apparatus having a first phase wavefront shape and a first power density distribution $F(\theta)$ in the first beam segment up to the angle $\theta_m$, and having a substantially spherical second phase wavefront shape and a substantially uniform second power density distribution of said beam as measured with respect to the angle $\beta$ in the second beam segment up to the angle $\beta_m$, said first means receiving one of said beam segments and being configured to be comprised of differential segments for redirecting by differing amounts the portions of received beam power received upon the segments at differing distances from said axis and cooperating with said offset distance to convert the power density distribution within said beam to the distribution in the other of said beam segments, said second boundary being configured to be comprised of a plurality of differential segments for redirecting by differing amounts the portions of beam power received upon segments at differing distances from said axis to follow lines emanating at angles $\theta$ and $\beta$ respectively from one of said center and focus, and wherein the portions of beam power flowing at any directional angle $\theta$ in the first segment flow at a respective directional angle $\beta$ in accordance with the relationship

$$\frac{d\beta}{d\theta} = \frac{(1-\cos\beta_m)F(\theta)\ \sin\theta}{\sin\beta \int_0^{\theta_m} F(\theta)\ \sin\theta d\theta}$$

* * * * *